ACIC - TC - 1679

TECHNICAL TRANSLATION

THE PROBLEM OF REDUCING THE FORMULA DESCRIBING THE SHAPE OF THE EARTH TO A TAYLOR SERIES

By: M.I. Marych

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Aeronautical Chart & Information Center	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

THE PROBLEM OF REDUCING THE FORMULA DESCRIBING THE SHAPE OF THE EARTH TO A TAYLOR SERIES.

4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Source: Geodeziya, Kartografiya i Aerofotos"yemka, Vol. 9, 1969, pp. 29-32, Mezhvedomstvennyy Respublikanskiy Nauchno-tekhnicheskiy Sbornik, Lvov, Ukrainian SSR.

5. AUTHOR(S) (First name, middle initial, last name)

Marych, M. I.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
November 1970	6	2

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	ACIC-TC-1679
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY This document has been approved for public release and sale; its distribution is unlimited.

13. ABSTRACT

Molodenskiy has suggested a new method of solving the integral equation for the density of a simple layer and has obtained formulae for successive approximations of the disturbing potential on the physical surface of the earth. If the earth's physical surface on which the gravity anomalies are given is assumed to be a sphere, then the disturbing potential will be determined by Stokes' formula. This paper shows an attempt to obtain this result using the Molodenskiy method of successive approximations, and also the method of transformations by expanding the disturbing potential in a Taylor series.

DD FORM 1 NOV 65 1473

Unclassified
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Figure of the earth						
Approximations						
Disturbing potential						
Gravity anomalies						
Stokes formula						
Taylors series						

TRANSLATION ACIC-TC- 1670

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(K voprosu privedeniya formuly, opredelyayushchey figuru Zemli, k ryadu Teylora)

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Translated by: George Synenko, Technical Translation Unit, November, 1970

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THE PROBLEM OF REDUCING THE FORMULA DESCRIBING THE SHAPE OF THE EARTH TO A TAYLOR SERIES

M.S. Molodenskiy has suggested a new method of solving his integral equation for the density of a simple layer and has obtained formulae for successive approximations of the disturbing potential on the physical surface of the earth /2/. Two first approximations, as is shown in work /1/, are reduced to the sum of the first two terms of a Taylor-series expansion of the disturbing potential in powers of the earth relief height, where the vertical gradients of the gravity anomalies are calculated according to Numerov's formula. If we assume the earth's physical surface on which the gravity anomalies are given to be a sphere, then the disturbing potential, as is known, will be determined by Stokes' formula. Let us try to obtain this result, using the Molodenskiy method of successive approximations, and also the method of transformations by expanding the disturbing potential in a Taylor series.

1. Let us find by Molodenskiy's method the values of the disturbing potential on the sphere of radius $\rho=a+H$, where a is the mean radius of the earth. First of all let us note that with H = const Molodenskiy's formula coincides with Stokes' formula. However, the integration here is carried out not with respect to the sphere of radius ρ, but with respect to the sphere of radius a. This is explained by the fact that in deriving the formula for the general case, small quantities depending on H were discarded. Consequently, let us proceed from the integral equation

$$2\pi\varphi = \Delta g + \frac{3}{2}\int \varphi \frac{d\sigma}{r},$$

where Δg = gravity anomaly;

$r = 2\sin\frac{\psi}{2}$;

ψ = angle formed by the radius vectors of the given and current points;
$d\sigma$ = surface element of the unit-radius sphere, density of simple layer distributed on the sphere of radius ρ.

Introducing the new function

$$\chi = \frac{\rho^2}{a^2}\varphi,$$

we have

$$2\pi\chi = \frac{\rho^2}{a^2}\Delta g + \frac{3}{2}\int \chi \frac{d\sigma}{r}.$$

By means of the small parameter $k (0 \leq k \leq 1)$ we may write

$$2\pi \sum_{n=0}^{\infty} k^n \chi_n = \frac{\bar{\rho}^2}{a^2} \Delta g + \frac{3}{2} \int \sum_{n=0}^{\infty} k^n \chi_n \frac{d\sigma}{r},$$

where $\bar{\rho} = a + kH$. The series entering into the left and right parts of the equation must be equal for all values of k : therefore the multipliers with k^n in both parts of the equation must be mutually equal. Thus, we obtain

$$2\pi\chi_n - \frac{3}{2} \int \chi_n \frac{d\sigma}{r} = G_n,$$

where

$$G_0 = \Delta g, \ G_1 = \frac{2H}{a} \Delta g, \quad G_2 = \frac{H^2}{a^2} \Delta g, \ G_{n>3} = 0.$$

Solving these integral equations, we find

$$\chi_n = \frac{G_n}{2\pi} + \frac{3}{(4\pi)^2} \int G_n s(\psi) \, d\sigma,$$

and also

$$\int \chi_n \frac{d\sigma}{r} = \frac{1}{4\pi} \int G_n s(\psi) \, d\sigma, \tag{1}$$

where $s(\psi)$ is the Stokes function. The disturbing potential determined by the formula

$$T = \frac{a^2}{\rho} \int \chi \frac{d\sigma}{r}, \tag{2}$$

is found, as well as the subsidiary function χ, by successive approximations, i.e.,

$$T = \sum_{n=0}^{\infty} T_n,$$

where each term T_n contains the height H only to the degree n. Thus, taking into consideration expressions (1) and (2) we obtain

$$T_0 = a\int \chi_0 \frac{d\sigma}{r} = \frac{a}{4\pi}\int \Delta g\, s(\psi)\, d\sigma;$$

$$T_1 = a\int \chi_1 \frac{d\sigma}{r} - H\int \chi_0 \frac{d\sigma}{r} = \frac{H}{4\pi}\int \Delta g\, s(\psi)\, d\sigma;$$

$$T_2 = a\int \chi_2 \frac{d\sigma}{r} - H\int \chi_1 \frac{d\sigma}{r} + \frac{H^2}{a}\int \chi_0 \frac{d\sigma}{r} = 0;$$

$$T_{n>3} = 0.$$

Consequently

$$T = \frac{\rho}{4\pi}\int \Delta g\, s(\psi)\, d\sigma. \tag{3}$$

Thus, if the anomalies Δg are given on the sphere, then Molodenskiy's process of successive approximations leads to Stokes' formula.

2. Let us use the expansion of the disturbing potential in a Taylor series

$$T = \frac{a}{4\pi}\int\left(\Delta g - H\frac{d\Delta g}{d\rho} + \frac{H^2}{2}\frac{d^2\Delta g}{d\rho^2} - \cdots\right)s(\psi)\, d\sigma + \frac{dT}{d\rho}H - \frac{1}{2}\frac{d^2T}{d\rho^2}H^2 + \cdots \tag{4}$$

and find the values T at points on the sphere of radius ρ. First let us obtain auxiliary relations. For this purpose we take into consideration Molodenskiy's integral formula

$$\frac{dU_0}{d\rho} = \frac{1}{2\pi\rho}\int \frac{U - U_0}{r^3}\, d\sigma + \frac{U_0}{\rho},$$

which determines the value of the normal derivative for the harmonic function U, given on the surface of the sphere /2/. Applying this formula to the harmonic functions

$$\rho\frac{dT}{d\rho},\ \rho\frac{d}{d\rho}\left(\rho\frac{dT}{d\rho}\right), \ldots,$$

where

$$\frac{dT}{d\rho} = -\Delta g - \frac{2T}{\rho}, \tag{5}$$

we find

$$\frac{d^i \Delta g}{d\rho^i} = \frac{1}{2\pi\rho}\int\left[\frac{d^{i-1}\Delta g}{d\rho^{i-1}} - \left(\frac{d^{i-1}\Delta g}{d\rho^{i-1}}\right)_0\right]\frac{d\sigma}{r^3} - (i+1)\frac{d^{i-1}\Delta g}{\rho d\rho^{i-1}}. \tag{6}$$

$$(i = 1, 2, \ldots)$$

Presenting the quantity $\frac{d^{i-1}\Delta g}{d\rho^{i-1}}$ in the form of a series expansion in spherical harmonics and considering that

$$r^{-3} = -\frac{1}{2}\sum_{n=0}^{\infty}(2n+1)(n+1)P_n(\psi),$$

where P_n are Legendre's polynomials of the nth order, after corresponding transformations we obtain

$$\frac{d^i \Delta g}{d\rho^i} = -\frac{1}{\rho}\sum_{n=0}^{\infty}(n+i+1)\left(\frac{d^{i-1}\Delta g}{d\rho^{i-1}}\right)_n. \tag{7}$$

These relations, and also

$$s(\psi) = \sum_{n=2}^{\infty}\frac{2n+1}{n-1}P_n(\psi) \tag{8}$$

make it possible without particular difficulties to carry out the integration indicated in formula (4). Since

$$\frac{a}{4\pi}\int\frac{d\Delta g}{d\rho}s(\psi)\,d\sigma = \frac{\rho}{4\pi}\int\frac{d\Delta g}{d\rho}s(\psi)\,d\sigma - \frac{H}{4\pi}\int\frac{d\Delta g}{d\rho}s(\psi)\,d\sigma,$$

then with the help of (7) and (8) we find

$$\frac{a}{4\pi}\int\frac{d\Delta g}{d\rho}s(\psi)\,d\sigma = -\Delta g - \frac{3T}{\rho} + \frac{H}{\rho}\left(\Delta g + \frac{3T}{\rho}\right), \tag{9}$$

where T is determined by Stokes' formula (3). In the same way we find

$$\frac{a}{4\pi}\int\frac{d^2\Delta g}{d\rho^2}s(\psi)\,d\sigma = -\frac{d\Delta g}{d\rho} + \frac{4\Delta g}{\rho} + \frac{12T}{\rho^2} + \frac{H}{\rho}\left(\frac{d\Delta g}{d\rho} - \frac{4\Delta g}{\rho} - \frac{12T}{\rho^2}\right). \tag{10}$$

Differentiating function (3) in the direction of ρ, we have

$$\frac{d^2T}{d\rho^2} = -\frac{d\Delta g}{d\rho} + \frac{2\Delta g}{\rho} + \frac{6T}{\rho^2}. \tag{11}$$

Let us now perform the calculations according to formula (4). With the help of expressions (5) and (9) we obtain

$$\frac{a}{4\pi}\int\left(\Delta g - H\frac{d\Delta g}{d\rho}\right)s(\psi)\,d\sigma + \frac{dT}{d\rho}H = T + \delta_2,$$

where

$$\delta_2 = -\frac{H^2}{\rho}\left(\Delta g + \frac{3T}{\rho}\right).$$

The given result differs from the exact value T of the disturbing potential by the quantity δ_2 containing H^2. If in formula (6) with i = 1 it is assumed $\rho = a$, which corresponds to the calculation of the vertical gradient of the gravity anomaly according to Numerov's formula, then instead of formula (9) we obtain

$$\frac{a}{4\pi}\int\frac{d\Delta g}{d\rho}s(\psi)\,d\sigma = -\Delta g - \frac{3T}{\rho},$$

and consequently the quantity δ_2 will not appear in the result. This fully agrees with the condition that the two first Molodenskiy approximations consider quantities which contain H only to the first degree. Further, taking into consideration (10) and (11) we have

$$\frac{a}{4\pi}\int\left(\Delta g - H\frac{d\Delta g}{d\rho} + \frac{H^2}{2}\frac{d^2\Delta g}{d\rho^2}\right)s(\psi)\,d\sigma + \frac{dT}{d\rho}H - \frac{1}{2}\frac{d^2T}{d\rho^2}H^2 = T + \delta_3,$$

where

$$\delta_3 = \frac{H^3}{\rho}\left(\frac{1}{2}\frac{d\Delta g}{d\rho} - \frac{2\Delta g}{\rho} - \frac{6T}{\rho^2}\right).$$

It is easily understood that the quantity δ_3 containing H^3 will not appear in the final result, if in formula (6) with i = 2 we set $\rho = a$.

In conclusion, let us note that the process of calculations according to formula (4), as well as Molodenskiy's process of successive approximations, leads to accurate values of the disturbing potential on the sphere of radius ρ, if only in the last of the used formulae (6) we assume $\rho = a$.

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